The null hypothesis for the runs test, \( H\_0: F(z) = G(z) \), where \( F(z) \) and \( G(z) \) are the cumulative distribution functions (CDFs) of two different distributions, essentially states that the two distributions from which the data are drawn are identical. Here's why this is equivalent to testing whether the data are randomly sampled:

1. \*\*Random Sampling Implication\*\*: When data are randomly sampled from a single population, the observed sequence of data points should not exhibit any systematic patterns or trends that would suggest non-randomness. This means that if we were to split the data into two groups (for instance, based on some characteristic or simply for the purpose of the test), the distribution of values in these two groups should be the same under random sampling.

2. \*\*CDF Equality\*\*: The statement \( F(z) = G(z) \) implies that at any value \( z \), the probability that a randomly chosen value from the first group (with CDF \( F \)) is less than or equal to \( z \) is the same as the probability that a randomly chosen value from the second group (with CDF \( G \)) is less than or equal to \( z \). This equality of CDFs means that the two groups are identically distributed, which is what we would expect if the data were randomly sampled from the same population.

3. \*\*Runs Test Context\*\*: The runs test looks for runs, which are sequences of similar events (like consecutive high or low values) in a dataset. Under \( H\_0 \), these runs should occur randomly without any predictable pattern if the data are indeed randomly sampled. If \( F(z) = G(z) \), any observed pattern in the sequence of data (like too many or too few runs) would suggest that the data do not follow a random pattern, contradicting the assumption of random sampling.

4. \*\*Non-Randomness Detection\*\*: If \( F(z) \neq G(z) \), it would suggest that there is some underlying structure or bias in how the data were sampled or grouped, leading to different distributions. This would be evidence against the hypothesis of random sampling, as random sampling should not produce such differences.

In summary, testing \( H\_0: F(z) = G(z) \) in the runs test is a way to check if the data sequence could have arisen from a random process without any systematic bias or pattern. If the null hypothesis holds, it supports the idea that the data are randomly sampled from the same distribution, showing no significant deviation from randomness. If it does not hold, it suggests non-randomness in the sampling or grouping process.